MA206X AY26-2 — Block II Practice Review

Statistical Inference (Lessons 17–25)

This review is for practice only and is not graded.

Work each problem completely. You are authorized your WPR2 SRC and R-lite interpreter (`qt`, `pt`, `qnorm`, `pnorm`). State hypotheses using population parameters, show all work for test statistics, report p-values using R-lite commands, and write conclusions in context.

Concept Check

Answer each of the following. These are quick checks of key definitions and properties.

1. True or False: The Central Limit Theorem requires the population to be normally distributed.

   Solution: False. The CLT says $\bar{X}$ is approximately normal for large n **regardless** of the population distribution.

2. True or False: A 95% confidence interval means there is a 95% probability that the true parameter lies within the interval.

   Solution: False. The correct interpretation is: if we repeated this process many times, 95% of the intervals would contain the true parameter. The parameter is fixed; the interval is random.

3. Fill in the blank: The standard error of $\bar{X}$ is ________________.

   Solution: $\sigma/\sqrt{n}$ (estimated by $s/\sqrt{n}$)

4. True or False: Failing to reject H_0 proves that the null hypothesis is true.

   Solution: False. Failing to reject means we lack sufficient evidence against H_0. Absence of evidence is not evidence of absence.

5. Fill in the blank: A Type I error occurs when we ______________________.

   Solution: Reject H_0 when H_0 is actually true (false alarm). $P(\text{Type I}) = \alpha$.

6. True or False: If a 90% CI for μ is $(52, 68)$, we would reject $H_0 : \mu = 70$ at $\alpha = 0.10$.

   Solution: True. Since $70 \notin (52, 68)$, we reject H_0 at $\alpha = 0.10$ by CI–HT duality.

7. Fill in the blank: For a one-proportion z-test, the success–failure condition requires ________________.

   Solution: $np_0 \geq 10$ and $n(1 - p_0) \geq 10$

8. True or False: In a two-proportion hypothesis test, we use the pooled proportion $\hat{p} = \frac{x_1+x_2}{n_1+n_2}$ in the standard error.

   Solution: True. Under $H_0 : p_1 = p_2$, we pool the data to estimate the common proportion. (For the CI, we use the individual $\hat{p}_i$'s instead.)

9. True or False: A paired t-test is appropriate when comparing two independent groups.

   Solution: False. A paired t-test requires naturally paired observations (same subjects, before/after, matched pairs). Independent groups use a two-sample t-test.

10. Fill in the blank: For a two-sample t-test (by hand), we use the conservative degrees of freedom $df =$ __________________.

    Solution: $\min(n_1 - 1,\ n_2 - 1)$

CLT & Sampling Distributions (Lesson 17)

Problem 1. A coffee shop's transaction times have a mean of $\mu = 4.5$ minutes and standard deviation $\sigma = 1.2$ minutes. A random sample of $n = 36$ transactions is selected.

(a) Describe the sampling distribution of $\bar{X}$.

Solution: By the CLT ($n = 36 \geq 30$), $\bar{X} \sim N\left(4.5,\ \frac{1.2^2}{36}\right) = N(4.5,\ 0.04)$. The standard error is $SE = 1.2/\sqrt{36} = 0.20$ minutes.

(b) Find $P(\bar{X} > 4.8)$.

Solution: $z = \frac{4.8-4.5}{0.20} = 1.50$. $P(\bar{X} > 4.8) = 1 - \texttt{pnorm(1.50)} = 1 - 0.9332 = 0.0668$.

(c) Find $P(4.3 < \bar{X} < 4.7)$.

Solution: $z_1 = \frac{4.3-4.5}{0.20} = -1.00$, $z_2 = \frac{4.7-4.5}{0.20} = 1.00$.
$P(4.3 < \bar{X} < 4.7) = \texttt{pnorm(1.00)} - \texttt{pnorm(-1.00)} = 0.8413 - 0.1587 = 0.683$.

Problem 2. A survey finds that 60% of consumers in a large city prefer online shopping. A random sample of $n = 200$ consumers is selected.

(a) Verify the CLT conditions for $\hat{p}$ and describe the sampling distribution.

Solution: $np = 200(0.60) = 120 \geq 10$ ✓ $n(1-p) = 200(0.40) = 80 \geq 10$ ✓
$\hat{p} \sim N\left(0.60,\ \frac{0.60(0.40)}{200}\right) = N(0.60,\ 0.0012)$. $SE = \sqrt{0.0012} = 0.0346$.

(b) Find $P(\hat{p} > 0.65)$.

Solution: $z = \frac{0.65-0.60}{0.0346} = 1.44$. $P(\hat{p} > 0.65) = 1 - \texttt{pnorm(1.44)} = 1 - 0.925 = 0.0749$.

(c) How large a sample would be needed so that the standard error of $\hat{p}$ is at most 0.02?

Solution: $SE = \sqrt{\frac{p(1-p)}{n}} \leq 0.02 \implies n \geq \frac{0.60(0.40)}{0.02^2} = \frac{0.24}{0.0004} = 600$.

Confidence Intervals (Lessons 18–19)

Problem 3. A bakery wants to estimate the average weight of its sourdough loaves. A random sample of 16 loaves yields $\bar{x} = 510$ grams and $s = 12$ grams. Assume loaf weights are approximately normal.

(a) Construct a 95% confidence interval for the true mean loaf weight.

Solution: $t_{0.025,15} = \texttt{qt(0.975, 15)} = 2.131$.
$\bar{x} \pm t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}} = 510 \pm 2.131 \cdot \frac{12}{\sqrt{16}} = 510 \pm 2.131(3) = 510 \pm 6.39$.
95% CI: $(503.6,\ 516.4)$ grams.

(b) Interpret the interval in context.

Solution: We are 95% confident that the true average weight of sourdough loaves from this bakery is between 503.6 and 516.4 grams.

(c) If the bakery wants the margin of error to be at most 3 grams (at 95% confidence), how many loaves should they sample?

Solution: Using the z approximation: $n = \left(\frac{z_{\alpha/2} \cdot s}{E}\right)^2 = \left(\frac{1.960 \cdot 12}{3}\right)^2 = (7.84)^2 = 61.5$.
Round up: $n = 62$ loaves.

Problem 4. A university surveys 500 randomly selected students and finds that 340 use the campus library at least once per week.

(a) Construct a 90% confidence interval for the true proportion of students who use the library weekly.

Solution: $\hat{p} = 340/500 = 0.680$. $z_{0.05} = \texttt{qnorm(0.95)} = 1.645$.
$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.680 \pm 1.645\sqrt{\frac{0.680(0.320)}{500}} = 0.680 \pm 1.645(0.02086) = 0.680 \pm 0.0343$.
90% CI: $(0.646,\ 0.714)$.

(b) Interpret the interval in context.

Solution: We are 90% confident that the true proportion of students who use the campus library at least once per week is between 64.6% and 71.4%.

(c) Without computing, would a 99% CI be wider or narrower? Explain.

Solution: Wider. A higher confidence level requires a larger critical value ($z_{0.005} = 2.576 > 1.645$), which increases the margin of error and widens the interval.

One-Sample t-Test (Lesson 21)

Problem 5. Customers at a restaurant complain that wait times exceed the advertised average of 20 minutes. A random sample of 25 visits yields $\bar{x} = 22.4$ minutes and $s = 5.1$ minutes.

(a) State the hypotheses to test whether the average wait time exceeds 20 minutes.

Solution: $H_0 : \mu = 20 \quad H_a : \mu > 20$

(b) At $\alpha = 0.05$, conduct the hypothesis test. Show the test statistic, p-value, decision, and conclusion.

Solution: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{22.4-20}{5.1/\sqrt{25}} = \frac{2.40}{1.02} = 2.35 \quad df = 24$

p-value = `1 - pt(2.35, 24)` $= 0.0138$

With a p-value of 0.0138 which is less than $\alpha = 0.05$, we reject H_0, meaning that the true average wait time exceeds 20 minutes.

(c) Construct a 95% confidence interval for μ and confirm it is consistent with your test decision.

Solution: $t_{0.025,24} =$ `qt(0.975, 24)` $= 2.064$

$22.4 \pm 2.064(1.02) = 22.4 \pm 2.11 = (20.3,\ 24.5)$

Since the entire interval is above 20, this is consistent with rejecting $H_0 : \mu = 20$ in favor of $\mu > 20$.

Problem 6. A laptop manufacturer advertises 8 hours of battery life. A consumer group tests 12 randomly selected laptops and finds $\bar{x} = 7.5$ hours with $s = 0.9$ hours. Test at $\alpha = 0.05$ whether the average battery life differs from the advertised claim.

(a) State the hypotheses.

Solution: $H_0 : \mu = 8 \quad H_a : \mu \neq 8$

(b) Conduct the test and state your conclusion.

Solution: $t = \frac{7.5-8}{0.9/\sqrt{12}} = \frac{-0.500}{0.260} = -1.92 \quad df = 11$

p-value = `2*(1 - pt(1.92, 11))` $= 2(0.0407) = 0.0814$

With a p-value of 0.0814 which is greater than $\alpha = 0.05$, we FTR H_0, meaning there is insufficient evidence that the true average battery life differs from 8 hours.

(c) Construct a 95% CI for the true mean battery life and verify consistency with your test.

Solution: $t_{0.025,11} =$ `qt(0.975, 11)` $= 2.201$

$7.5 \pm 2.201(0.260) = 7.5 \pm 0.572 = (6.93,\ 8.07)$

Since 8 is inside the interval, this is consistent with failing to reject H_0.

One-Proportion z-Test (Lesson 22)

Problem 7. An online retailer claims that at least 80% of orders are delivered on time. In a random sample of 400 orders, 300 arrived on time. Test the claim at $\alpha = 0.05$.

(a) State the hypotheses.

Solution: $H_0 : p = 0.80 \quad H_a : p < 0.80$

(b) Verify the conditions for the test.

Solution: $np_0 = 400(0.80) = 320 \geq 10$ ✓ $\quad n(1 - p_0) = 400(0.20) = 80 \geq 10$ ✓
Random sample stated. Conditions met.

(c) Conduct the test and state your conclusion.

Solution: $\hat{p} = 300/400 = 0.750$
$z = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{0.750-0.800}{\sqrt{0.80(0.20)/400}} = \frac{-0.0500}{0.0200} = -2.50$
p-value = `pnorm(-2.50)` = 0.00621
With a p-value of 0.00621 which is less than $\alpha = 0.05$, we reject H_0, meaning that the true on-time delivery rate is less than 80%.

(d) Construct a 95% confidence interval for the true on-time delivery rate.

Solution: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.750 \pm 1.960\sqrt{\frac{0.750(0.250)}{400}} = 0.750 \pm 1.960(0.02165) = 0.750 \pm 0.0424$
95% CI: (0.708, 0.792). Since 0.80 is not in this interval, it is consistent with rejecting the retailer's claim.

Problem 8. A university claims that at least 85% of its students graduate within four years. A sample of 500 recent students reveals that 405 graduated on time. Test at $\alpha = 0.01$.

(a) State hypotheses, conduct the test, and state your conclusion.

Solution: $H_0 : p = 0.85 \quad H_a : p < 0.85$
Check: $np_0 = 425 \geq 10$, $n(1 - p_0) = 75 \geq 10$ ✓
$\hat{p} = 405/500 = 0.810$
$z = \frac{0.810-0.850}{\sqrt{0.85(0.15)/500}} = \frac{-0.0400}{\sqrt{0.000255}} = \frac{-0.0400}{0.01597} = -2.50$
p-value = `pnorm(-2.50)` = 0.00621
With a p-value of 0.00621 which is less than $\alpha = 0.01$, we reject H_0, meaning that the true four-year graduation rate is less than 85%.

(b) Is the result practically significant? The university's accreditation requires at least 80%.

Solution: While statistically significant, the sample proportion of 81% is above the 80% accreditation threshold. The university is slightly below its own 85% claim but still meets the external requirement. This illustrates how statistical significance does not always imply practical concern.

Two-Sample t-Test (Lesson 23)

Problem 9. A university wants to compare weekly study hours between STEM and humanities majors. Independent random samples yield:

Group	n	$\bar{x}$	s
STEM	30	18.5 hrs	4.2 hrs
Humanities	35	15.8 hrs	3.6 hrs

(a) State the hypotheses to test whether average study hours differ between the two groups.

Solution: $H_0 : \mu_1 - \mu_2 = 0 \quad H_a : \mu_1 - \mu_2 \neq 0$
where μ_1 = true mean study hours for STEM, μ_2 = true mean study hours for humanities.

(b) Compute the test statistic, p-value, and state your conclusion at $\alpha = 0.05$.

Solution: $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{4.2^2}{30} + \frac{3.6^2}{35}} = \sqrt{0.588 + 0.370} = \sqrt{0.958} = 0.979$
$t = \frac{(18.5-15.8)-0}{0.979} = \frac{2.70}{0.979} = 2.76 \quad df = \min(29, 34) = 29$
p-value = `2*(1 - pt(2.76, 29))` = 2(0.00498) = 0.00996
With a p-value of 0.00996 which is less than $\alpha = 0.05$, we reject H_0, meaning that there is a significant difference in average weekly study hours between STEM and humanities majors.

(c) Construct a 95% CI for $\mu_1 - \mu_2$.

Solution: $t_{0.025,29}$ = `qt(0.975, 29)` = 2.045
$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2,df} \cdot SE = 2.70 \pm 2.045(0.979) = 2.70 \pm 2.00 = (0.698,\ 4.70)$
We are 95% confident that STEM majors study between 0.7 and 4.7 more hours per week than humanities majors. Since 0 is not in the interval, this is consistent with rejecting H_0.

Problem 10. A transportation analyst compares average commute times (in minutes) between two cities:

City	n	$\bar{x}$	s
City A	40	32.5	8.3
City B	45	28.1	7.6

(a) Test at $\alpha = 0.05$ whether City A residents have a longer average commute than City B residents.

Solution: $H_0 : \mu_A - \mu_B = 0 \quad H_a : \mu_A - \mu_B > 0$
$SE = \sqrt{\frac{8.3^2}{40} + \frac{7.6^2}{45}} = \sqrt{1.722 + 1.284} = \sqrt{3.006} = 1.734$
$t = \frac{32.5 - 28.1}{1.734} = \frac{4.40}{1.734} = 2.54 \quad df = \min(39, 44) = 39$
p-value = `1 - pt(2.54, 39)` = 0.00764
With a p-value of 0.00764 which is less than $\alpha = 0.05$, we reject H_0, meaning that City A residents have a significantly longer average commute than City B residents.

(b) Construct a 95% CI for the difference in mean commute times.

Solution: $t_{0.025,39}$ = `qt(0.975, 39)` = 2.023
$4.40 \pm 2.023(1.734) = 4.40 \pm 3.51 = (0.893,\ 7.91)$ minutes.
We are 95% confident that City A residents commute between 0.9 and 7.9 minutes longer, on average, than City B residents.

Paired t-Test (Lesson 24)

Problem 11. A company wants to evaluate whether ergonomic keyboards improve typing speed. Eight employees are tested before and after switching to the new keyboards. The differences (after − before) yield $\bar{d} = 5.6$ wpm, $s_d = 3.8$ wpm.

(a) Explain why a paired t-test is appropriate here rather than a two-sample t-test.

Solution: The same 8 employees are measured twice (before and after), creating natural pairs. The observations within each pair are not independent—an employee's "before" and "after" scores are correlated. A paired t-test accounts for this by analyzing the differences.

(b) Test at $\alpha = 0.05$ whether the new keyboards increased typing speed.

Solution: $H_0 : \mu_d = 0 \quad H_a : \mu_d > 0 \quad$ (where d = after − before)
$t = \frac{\bar{d}-0}{s_d/\sqrt{n}} = \frac{5.6}{3.8/\sqrt{8}} = \frac{5.6}{1.344} = 4.17 \quad df = 7$
p-value = `1 - pt(4.17, 7)` = 0.00210
With a p-value of 0.00210 which is less than $\alpha = 0.05$, we reject H_0, meaning that the ergonomic keyboards significantly increased typing speed.

(c) Construct a 95% CI for the true mean improvement in typing speed.

Solution: $t_{0.025,7}$ = `qt(0.975, 7)` = 2.365
$5.6 \pm 2.365(1.344) = 5.6 \pm 3.18 = (2.42,\ 8.78)$ wpm.
We are 95% confident that the true mean improvement in typing speed is between 2.4 and 8.8 words per minute.

Problem 12. A counseling center tests whether a new meditation program reduces student anxiety scores. Ten students are measured before and after the 6-week program. The differences (before − after) yield $\bar{d} = 3.2$ points, $s_d = 6.5$ points.

(a) Test at $\alpha = 0.05$ whether the program reduced anxiety scores.

Solution: $H_0 : \mu_d = 0 \quad H_a : \mu_d > 0 \quad$ (where d = before − after, so positive d means reduction)
$t = \frac{3.2}{6.5/\sqrt{10}} = \frac{3.2}{2.055} = 1.56 \quad df = 9$
p-value = `1 - pt(1.56, 9)` = 0.0770
With a p-value of 0.0770 which is greater than $\alpha = 0.05$, we FTR H_0, meaning there is insufficient evidence that the meditation program reduced anxiety scores.

(b) Construct a 95% CI for μ_d and verify consistency.

Solution: $t_{0.025,9}$ = `qt(0.975, 9)` = 2.262
$3.2 \pm 2.262(2.055) = 3.2 \pm 4.65 = (-1.45,\ 7.85)$
Since 0 is in the interval, it is plausible that there is no reduction—consistent with failing to reject.

Two-Proportion z-Test (Lesson 25)

Problem 13. Two clinics compare patient satisfaction rates. Clinic A surveyed 250 patients and found 180 satisfied. Clinic B surveyed 200 patients and found 120 satisfied. Test at $\alpha = 0.05$ whether the satisfaction rates differ.

(a) State the hypotheses.

Solution: $H_0 : p_1 - p_2 = 0 \quad H_a : p_1 - p_2 \neq 0$
where p_1 = true satisfaction rate at Clinic A, p_2 = true satisfaction rate at Clinic B.

(b) Conduct the test.

Solution: $\hat{p}_1 = 180/250 = 0.720, \quad \hat{p}_2 = 120/200 = 0.600$
$\hat{p} = \frac{180+120}{250+200} = \frac{300}{450} = 0.667$ (pooled under H_0)
$SE = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{0.667(0.333)(0.009)} = \sqrt{0.00200} = 0.0447$
$z = \frac{0.720-0.600}{0.0447} = \frac{0.120}{0.0447} = 2.68$
p-value = `2*(1 - pnorm(2.68))` $= 2(0.00368) = 0.00736$
With a p-value of 0.00736 which is less than $\alpha = 0.05$, we reject H_0, meaning that the satisfaction rates differ significantly between the two clinics.

(c) Construct a 95% CI for $p_1 - p_2$.

Solution: For the CI, use **unpooled** standard error:
$SE_{CI} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = \sqrt{\frac{0.720(0.280)}{250} + \frac{0.600(0.400)}{200}} = \sqrt{0.000806 + 0.00120} = \sqrt{0.00201} = 0.0448$
$(\hat{p}_1 - \hat{p}_2) \pm 1.960 \cdot SE_{CI} = 0.120 \pm 1.960(0.0448) = 0.120 \pm 0.0878$
95% CI: $(0.0322, \ 0.208)$.
We are 95% confident that Clinic A's satisfaction rate is between 3.2 and 20.8 percentage points higher than Clinic B's.

Problem 14. Two coffee chains survey customers about a new seasonal blend. Chain A: 156 out of 240 prefer the new blend. Chain B: 120 out of 200 prefer the new blend. Test at $\alpha = 0.05$ whether the preference rates differ.

(a) Conduct the full hypothesis test.

Solution: $H_0 : p_1 - p_2 = 0 \quad H_a : p_1 - p_2 \neq 0$
$\hat{p}_1 = 156/240 = 0.650, \quad \hat{p}_2 = 120/200 = 0.600$
$\hat{p} = \frac{156+120}{240+200} = \frac{276}{440} = 0.627$
$SE = \sqrt{0.627(0.373)\left(\frac{1}{240} + \frac{1}{200}\right)} = \sqrt{0.234(0.00917)} = \sqrt{0.00214} = 0.0463$
$z = \frac{0.650-0.600}{0.0463} = \frac{0.0500}{0.0463} = 1.08$
p-value = `2*(1 - pnorm(1.08))` $= 2(0.140) = 0.280$
With a p-value of 0.280 which is greater than $\alpha = 0.05$, we FTR H_0, meaning there is insufficient evidence that the preference rates differ between the two chains.

(b) Construct a 95% CI for $p_1 - p_2$ and verify consistency.

Solution: $SE_{CI} = \sqrt{\frac{0.650(0.350)}{240} + \frac{0.600(0.400)}{200}} = \sqrt{0.000948 + 0.00120} = \sqrt{0.00215} = 0.0464$

$0.0500 \pm 1.960(0.0464) = 0.0500 \pm 0.0909 = (-0.0409,\ 0.141)$

Since 0 is in the interval, it is plausible there is no difference—consistent with FTR.

Mixed Practice

Problem 15. For each scenario, identify the most appropriate test and define the parameter of interest. You do **not** need to carry out the test.

(a) A hospital wants to know if the average patient stay differs from 4.5 days. They sample 30 patients.

Solution: One-sample t-test. Parameter: μ = true mean patient stay (days).

(b) A polling firm wants to test whether more than 55% of voters support a ballot measure. They poll 800 voters.

Solution: One-proportion z-test. Parameter: p = true proportion of voters who support the measure.

(c) A fitness center measures resting heart rate of 20 members before and after a 12-week exercise program.

Solution: Paired t-test. Parameter: μ_d = true mean difference in resting heart rate (before − after). Same subjects measured twice.

(d) An economist compares average household spending between two cities using independent samples of 50 households from each.

Solution: Two-sample t-test. Parameter: $\mu_1 - \mu_2$ = difference in true mean household spending.

(e) A researcher compares the proportion of defective items from two different factories.

Solution: Two-proportion z-test. Parameter: $p_1 - p_2$ = difference in true defect rates.

Problem 16. A food safety inspector tests $H_0 : \mu = 40°\text{F}$ (food is stored at proper temperature) vs. $H_a : \mu > 40°\text{F}$ (food is stored too warm) at $\alpha = 0.05$.

(a) Describe a Type I error in this context. What is its probability?

Solution: Type I error: Concluding that food is stored too warm when it is actually at the proper temperature. This would trigger unnecessary corrective actions. $P(\text{Type I}) = \alpha = 0.05$.

(b) Describe a Type II error in this context.

Solution: Type II error: Failing to detect that food is stored too warm when it actually is. This could lead to a food safety hazard going unaddressed. $P(\text{Type II}) = \beta$.

(c) Which error is more serious here? How could you reduce it?

Solution: Type II is more serious—missing a genuine food safety hazard. To reduce β (increase power): increase sample size n, increase α, or improve measurement precision (reduce σ).

Problem 17. A 95% confidence interval for μ is $(48.2,\ 55.8)$.

(a) Would you reject $H_0 : \mu = 46$ vs. $H_a : \mu \neq 46$ at $\alpha = 0.05$? Explain.

Solution: Yes. $46 \notin (48.2, 55.8)$, so reject H_0 at $\alpha = 0.05$ by CI–HT duality.

(b) Would you reject $H_0 : \mu = 50$ vs. $H_a : \mu \neq 50$ at $\alpha = 0.05$? Explain.

Solution: No. $50 \in (48.2, 55.8)$, so FTR H_0 at $\alpha = 0.05$.

(c) Can you determine whether you would reject $H_0 : \mu = 46$ at $\alpha = 0.01$? Explain.

Solution: Not from this interval alone. A 95% CI corresponds to $\alpha = 0.05$. To test at $\alpha = 0.01$, we would need a 99% CI (which would be wider). Since 46 is outside the 95% CI, it might or might not be outside a 99% CI—we cannot determine this without more information.

Problem 18. A grocery store claims the average checkout time is at most 7 minutes. A random sample of 20 transactions yields $\bar{x} = 8.2$ minutes and $s = 2.4$ minutes. Test at $\alpha = 0.05$.

(a) Conduct the full hypothesis test.

Solution: $H_0 : \mu = 7 \quad H_a : \mu > 7$
$t = \frac{8.2-7}{2.4/\sqrt{20}} = \frac{1.20}{0.537} = 2.24 \quad df = 19$
p-value = `1 - pt(2.24, 19)` $= 0.0188$
With a p-value of 0.0188 which is less than $\alpha = 0.05$, we reject H_0, meaning that the average checkout time exceeds 7 minutes.

(b) Construct a 90% CI for the true average checkout time.

Solution: $t_{0.05,19}$ = `qt(0.95, 19)` $= 1.729$
$8.2 \pm 1.729(0.537) = 8.2 \pm 0.928 = (7.27,\ 9.13)$ minutes.
We are 90% confident the true average checkout time is between 7.27 and 9.13 minutes.

Problem 19. A tutoring company tests whether its program improves student exam scores. Fifteen students are tested before and after the program. Summary statistics for the differences (after − before): $\bar{d} = 4.8$ points, $s_d = 7.2$ points.

(a) Identify the appropriate test and justify your choice.

Solution: Paired t-test. The same 15 students are measured before and after—observations are paired by student, not independent.

(b) Conduct the test at $\alpha = 0.05$.

Solution: $H_0 : \mu_d = 0 \quad H_a : \mu_d > 0$
$t = \frac{4.8}{7.2/\sqrt{15}} = \frac{4.8}{1.859} = 2.58 \quad df = 14$
p-value = `1 - pt(2.58, 14)` $= 0.0109$
With a p-value of 0.0109 which is less than $\alpha = 0.05$, we reject H_0, meaning that the tutoring program significantly improved exam scores.

(c) Construct a 95% CI for the true mean improvement.

Solution: $t_{0.025,14}$ = `qt(0.975, 14)` $= 2.145$
$4.8 \pm 2.145(1.859) = 4.8 \pm 3.99 = (0.813,\ 8.79)$ points.
We are 95% confident that the true mean improvement is between 0.8 and 8.8 points.

Problem 20. A large university ($n = 5000$ students) tests whether the average GPA differs from 3.00 and finds $\bar{x} = 3.02$ with $s = 0.45$.

(a) Compute the test statistic and p-value for $H_0 : \mu = 3.00$ vs. $H_a : \mu \neq 3.00$.

Solution: $t = \frac{3.02-3.00}{0.45/\sqrt{5000}} = \frac{0.0200}{0.00636} = 3.14 \quad df = 4999$
p-value = `2*(1 - pt(3.14, 4999))` ≈ 0.00170

(b) What is your decision at $\alpha = 0.05$?

Solution: With a p-value of 0.00170 which is less than $\alpha = 0.05$, we reject H_0, meaning that the true average GPA differs from 3.00.

(c) The approximate 95% CI is (3.008, 3.032). Is this result practically meaningful? Explain.

Solution: No. While statistically significant, the estimated difference is only 0.02 GPA points—far too small to matter in any practical context (e.g., admissions, academic standing, funding). The very large sample size ($n = 5000$) gave the test enough power to detect this trivially small difference.

(d) What feature of this problem drives the small p-value despite the tiny effect?

Solution: The extremely large sample size. With $n = 5000$, the standard error is tiny (0.00636), so even a difference of 0.02 GPA points produces a large test statistic. This is a classic example of statistical significance without practical significance.